1. Show that \( \int_{0}^{\infty} e^{-ax} J_0(bx^{1/2}) \, dx = \frac{1}{a} e^{-\frac{b^2}{4a}} \) by integrating the power series of \( J_0(bx^{1/2}) \) term by term.

2. The generating function in Equation 11 in Sec. 14.1 is a function of \( 2xt - t^2 \), which we can write as \( G(x, t) = F(2xt - t^2) \). First show that \( \frac{\partial G}{\partial x} = 2t \, F' \) and \( \frac{\partial G}{\partial t} = (2x - 2t) \, F' \), where \( F' \) means \( \frac{dF}{d(2xt - t^2)} \). Now show that \( (x - t) \frac{\partial G}{\partial x} - t \, \frac{\partial G}{\partial t} = 0 \). Given that

\[
G(x, t) = \sum_{n=0}^{\infty} P_n(x) t^n, \quad \text{show that the Legendre polynomials satisfy the differential recursion formula} \quad x \, P_n'(x) = n \, P_n(x) + P_{n-1}'(x).
\]

3. Expand \( f(x) = 1 - x^2 / 4, \quad -2 \leq x \leq 2 \), in terms of Legendre polynomials.

4. Starting with the recursion formula for Hermite polynomials in Table 14.2 in Sec. 14.2, derive the generating function \( G(x, t) = e^{2xt - t^2} = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n \). Let \( H_0(x) = 1 \).