Math 422/522 Homework 2

1. For each of the following vector functions determine whether the equation \( \nabla \varphi = \vec{F} \) possesses a solution, and determine that solution if it exists.
   (a) \( \vec{F} = 2xyz\hat{i} - (x^2 z + 2 y)\hat{j} + 3x^2 yz^2\hat{k} \)
   (b) \( \vec{F} = 2xyz\hat{i} + (x^2 + 2yz)\hat{j} + (y^3 + 1)\hat{k} \)

2. For each of the vector functions defined in problem 1, determine the value of the integral \( \int_C \vec{F} \cdot d\vec{r} \) from the origin to the point \((1,1,1)\) along the curve specified by the simultaneous equations \( y = x^4, z = x^3 \).

3. Evaluate the surface integral of the vector \( \vec{F} = x\hat{i} + y\hat{j} + zk \) over that portion of the surface \( z = xy + 1 \) which covers the square \( 0 \leq x \leq 1, 0 \leq y \leq 1 \) in the xy plane.

4. Determine the value of the surface integral \( \iint_S \vec{F} \cdot \hat{n} \, d\sigma \) in each of the following cases by use of the Divergence theorem:
   (a) \( \vec{F} = x\hat{i} + y\hat{j} + zk; \) \( S \) is the closed spherical surface \( x^2 + y^2 + z^2 = 1 \).
   (b) \( \vec{F} = xy\hat{i} + xz\hat{j} + (1 - z - yz)\hat{k}; \) \( S \) is the portion of the paraboloid \( z = 1 - x^2 - y^2 \) for which \( z \geq 0 \).