1. Determine the solution of Laplace’s equation in the rectangle \(0 \leq x \leq l, 0 \leq y \leq d\) which satisfies the conditions \(T(0, y) = T(l, y) = 0, \quad T(x, 0) = g(x), \quad T(x, d) = f(x)\).

[Suggestion: Notice that a convenient form of a particular solution of the equation, satisfying the homogeneous conditions, is \(T_p = \left[ a_i \sinh \frac{n \pi y}{l} + b_i \sin \frac{n \pi (d-y)}{l} \right] \sin \frac{n \pi x}{l}\), where \(n\) is a positive integer.]

2. The rectangular plate discussed in class has edges at \(x = 0, x = l, y = 0\) and \(y = d\).

The boundary conditions on each edge were given by

\(T(0, l) = 0, \quad T(l, y) = 0, \quad T(x, 0) = 0\) and \(T(x, d) = f(x)\). Suppose that the plate is now of infinite extent in the \(y\) direction, on one side of the boundary \(y = 0\), so that it occupies the semi-infinite strip \(0 \leq x \leq l, 0 \leq y < \infty\).

If the temperature is to vanish on the lateral boundaries \(x = 0\) and \(x = l\),

is to tend to zero as \(y \to \infty\), and is to reduce to \(f(x)\) along the edge \(y = 0\),

obtain the temperature distribution in the form

\[T(x, y) = \sum_{n=1}^{\infty} c_n e^{-\frac{n \pi}{l} x} \frac{n \pi x}{l} f(x) \sin \frac{n \pi y}{l}, \quad c_n = 2 \int_0^l f(x) \sin \frac{n \pi x}{l} dx.\]

3. Let the previous problem be modified in such a way that the rate of heat flow, per unit distance, into the plate through points of the boundary \(y = 0\) is prescribed as \(g(x)\), where \(g(x)\) may be measured in calories per second per centimeter length along the boundary \(y = 0\).

(a) Show that the condition along the line \(y = 0\) then is of the form

\[-k h \frac{\partial T(x, 0)}{\partial y} = g(x),\]

where \(h\) is the thickness of the plate.

(b) Obtain the solution in the form

\[T(x, y) = \sum_{n=1}^{\infty} c_n e^{-\frac{n \pi}{l} x} \frac{n \pi y}{l} g(x) \sin \frac{n \pi y}{l} dx, \quad c_n = \frac{2}{n \pi kh} \int_0^l g(x) \sin \frac{n \pi x}{l} dx.\]