1. If $F$ is a function of $t$, find the derivative of

$$F \cdot \frac{dF}{dt} \times \frac{d^2F}{dt^2}$$

2. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ represents the position vector from a fixed origin $O$ to a point $P$, and suppose that the $xyz$ axis system is rotating about a fixed vector $\omega$ through $O$, with angular velocity of constant magnitude $\omega$.

(a) By calculating $\frac{dr}{dt}$, and noticing that $\frac{dl}{dt} = \omega \times \mathbf{i}$, and so forth, obtain the velocity vector in the form

$$\mathbf{v} = \mathbf{v}_O + \omega \times \mathbf{r}$$

where the vector

$$\mathbf{v}_O = \frac{dx}{dt} \mathbf{i} + \frac{dy}{dt} \mathbf{j} + \frac{dz}{dt} \mathbf{k}$$

is the velocity vector which would be obtained if the axes were fixed.

(b) Obtain the acceleration vector in the form

$$\mathbf{a} = \mathbf{a}_O + 2\omega \times \mathbf{v}_O + \omega \times (\omega \times \mathbf{r}),$$

Where $\mathbf{v}_O$ is defined in part (a), and where

$$\mathbf{a}_O = \frac{d^2x}{dt^2} \mathbf{i} + \frac{d^2y}{dt^2} \mathbf{j} + \frac{d^2z}{dt^2} \mathbf{k}$$

3. Determine the unit vector normal to the surface $x^3 - xyz + z^3 = 1$ at the point $(1,1,1)$.

4. Show that $\nabla \cdot (x\mathbf{v}) = x\nabla \cdot \mathbf{v} + \mathbf{i} \cdot \mathbf{v}$ and $\nabla \times (x\mathbf{v}) = x\nabla \times \mathbf{v} + \mathbf{i} \times \mathbf{v}$
5. In class, we derived an expression for the acceleration vector of a body moving along a space curve in 3D. The latter can be decomposed into a tangential (unit vector $\hat{u}$) and normal component (unit vector $\hat{n}$). By defining a third binormal unit vector $\hat{b}$ perpendicular to the plane containing both $\hat{u}$ and $\hat{n}$, derive the Frenet formulas for the motion on an arbitrary space curve in 3D:

\[
\frac{d\hat{u}}{ds} = \frac{1}{\rho} \hat{n} \\
\frac{d\hat{b}}{ds} = -\frac{1}{\tau} \hat{n} \\
\frac{d\hat{n}}{ds} = \frac{1}{\tau} \hat{b} - \frac{1}{\rho} \hat{u}
\]

Where $s$ is the arclength, $\rho$ is the curvature and $\tau$ is the radius of torsion of the space curve. Derive explicit formulas for the reciprocal of $\rho$ and of $\tau$. Show full details of your derivation for full credit.